Optimal Passive Dynamics for Torque/Force Control

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***Abstract*— For robotic manipulation tasks in uncertain environments, good force control can provide significant benefits. The design of force-controlled actuators typically revolves around developing the best possible software control strategy. Discussions about the passive dynamics are often imprecise, lacking comprehensive details about the physical limitations. This paper develops relationships between an actuator's passive dynamics (stiffness, damping, motor inertia and torque limits) and the resulting performance, for the purpose of better understanding how to select the passive dynamics for a force-control task. We present two distinct scenarios for the actuator system and calculate the required input to produce the desired output. These exact solutions provide the basis for understanding how the parameters effect bandwidth. Our model does not include active control; we computed the optimal input to the system to produce the required torque at the load with zero error. This is important so that our results only reflect the physical system's performance.**

I. Introduction

Robots excel at precise position control and are useful for tasks that make use of this ability, such as CNC machining. However, physical interaction tasks such as catching a ball, walking, running, grasping unknown objects, constrained contact and even simple force control have historically been difficult for robots. Each of these tasks involve dynamic effects such as unexpected impacts and a significant transfer of kinetic energy between the robot and its environment. Animals far out perform robots at such tasks and we contend that this is due to inherent mechanical limitations in traditional robotic mechanisms, rather than software control inadequacies. This paper focuses on how an actuators passive dynamics affect force-control.

Consider a traditional industrial robot arm, powered by electric motors with large gear reductions and rigid links. The traditional approach to force control will utilize such an arm, with a force sensor placed at the end-effector. Forces are measured, software controllers calculate the desired motor torques and the motors move accordingly. However, the motors have inertia, which is amplified through the gearbox into a significant reflected inertia and combines with torque limitations on the motors to limit their acceleration. These passive dynamics cannot be overcome using software control. If an object impacts the arm, such as a baseball, the motors will have no chance to respond, the arm will behave as a rigid inertial object and the software control will have no part in its dynamic response.

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Passive dynamics are not always harmful. As an example of passive dynamics improving performance, a mechanical spring in series with a motor can dramatically improve force control bandwidth in response to position disturbances. However, this improvement applies only to the specific case of force control and its robustness to position disturbances; a series spring will reduce the performance of the system for position control. For peak performance in a robotic system, the passive dynamics must be tailored to the specific task. This is roughly analogous to impedance matching in electrical systems.

In this paper, we lay out a mathematical framework for mechanical systems that includes a motor with inertia and torque limts, a series spring and a series damper, as shown in Fig. 1. For two examples, applying constant force to a moving object and applying changing force to a stationary object, we describe the mathematically optimal passive dynamics required to achieve the best possible bandwidth, based on fundamental physical limits. Based on this work, robot designers will be able to guarantee that a mechanical system has the bandwidth necessary for a particular task, especially tasks involving force control, spring-like behavior, impacts and kinetic energy transfer.

II. Background

Muscular systems in animals incorporate elastic elements [1][2][4][7], which are most often examined while investigating locomotion, and are generally discussed in the context of energy storage. Roboticists have built machines designed to mimic this spring-like behavior [5][9]. Although the designers of these running machines acknowledge that elasticity provides robustness, their studies generally focus on energy storage and efficiency with little attention to force control.

Early investigations into force control found that compliance in an actuator can increase it's stability [15]. Later research found that active force control requires passive compliance [11]. Researchers at the Massachusetts Institute of Technology (MIT) Leg Laboratory explored the idea of an actuator designed specifically to include an elastic element as a force sensor and low impedance coupling between the drive system and the load. The system is aptly dubbed a series elastic actuator (SEA) and it has been shown that this configuration provides filtering to handle shock loads as well as higher resolution/bandwidth force control [8][10]. Despite showing how the system provides great advantages, only approximate guidelines for choosing an appropriate spring constant have been developed and presented. Further work to improve the design has focused on control architecture [16][13] or transmission design [14][12].

A similar actuator design using a viscous damper in place of the elastic element, dubbed a series damper actuator (SDA), has been proposed by Chew et al. [3]. They hypothesize that using damping, rather than elasticity, allows for greater bandwidth and can be easily constructed to allow a variable damping coefficient. They admit that the main disadvantage of the SDA is the energy dissipation property which limits energy economy of the design. They do not provide concrete relationships between damping and bandwidth, but present a conjecture relating the two.

A hybrid of the SDA and SEA has been proposed by Hurst et al. [6]. They concluded that the added damping provides higher bandwidth than a purely series-elastic element and reduces unwanted oscillations in specific situations. Initial force spikes observed by the drive system at impact are greater than would be observed by just an elastic element, but are still much less than for a perfectly stiff system.

III. SYSTEM MODEL

In this paper, we define the relationships between series stiffness, series damping, drive system inertia and the drive system torque limits in a specific experimental scenario. To simplify the discussion, we use “motor” to describe the drive system as a whole - transmission and motor characteristics. The following symbols describe our model:

ω	Angular frequency	$\frac{rad}{s}$
k	Spring constant	$\frac{N \cdot m}{rad}$
B	Damping constant	$\frac{kg \cdot m^2}{s \cdot rad}$
I_m	Motor inertia	$kg \cdot m^2$
τ_m	Motor torque	$N \cdot m$
τ_{limit}	Motor torque limit	$N \cdot m$
τ_L	Load torque	$N \cdot m$
θ_m	Motor angle	rad
θ_L	Load angle	rad
θ_A	Load angle amplitude	rad

Our goal in this paper is to calculate the fundamental limitations of the physical system. Our model does not include active control; we compute the optimal input to the system to produce a desired torque at the load. This is an important distinction from previous attempts to develop actuators of this nature. By eliminating controller error, we are able isolate the physical limitations of our model.

To develop the relationships between an actuator’s design parameters, we investigate the series elastic/damping actuator (SEDA) in Fig. 1. Our actuator includes damping and elasticity because they are both physically unavoidable and possibly useful. We want to know how to work with these elements (k, B and I_m) to design the best possible actuator around a force control task.

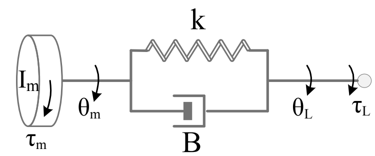


Fig. 1. The system we investigate in this paper is entirely rotational and includes damping, elasticity, motor inertia and torque limtis.

We arbitrarily chose our system model to be entirely rotational because our lab is interested in developing robots with electric motors. The concepts presented in this paper relate directly to force control just as well as torque control. Roboticists designing actuators with linear drive systems (such as hydraulic pistons) can use the relationships presented in this paper to develop linear systems.

In addition to the reactive elements k and B, we include motor torque limits as well as motor inertia. The torque limit and motor inertia are important for the calculation of the system bandwidth. If infinite torque were possible, there would be no requirements for designing the impedance of the actuator. In other words, it would not matter how soft, or stiff, the elements were, just as long as they existed. In the case of zero motor inertia with motor torque limits, the elastic or damping elements are no longer important. The elements just need to exist to provide the transmission of torque. In this case the largest torque the actuator could produce at the load would be the torque limit. In either case the system is optimal and has infinite bandwidth for any task while the impedance of the actuator is irrelevant. Unfortunately, this is not the case with real systems because all motors have torque limits and rotor inertia.

IV. ACTUATION SCENARIOS

Each scenario is designed to show that there is an optimal relationship between k, B and I_m for a distinct task. This paper focuses on simple, fundamental motions a force or torque controlled actuator might be expected to perform. The goal is to relate k, B, I_m and τ_{limit} under specific conditions to provide insight into the performance of a robotic actuator.

For the first task, our model applies a varying torque to a fixed load (Fig. 2). We demonstrate how k, B and I_m effect the maximum frequency at which the actuator can change the applied torque. The maximum frequency for this case is defined as the frequency that the actuator can oscillate the torque at the load before steady-state error is encountered.

The second task requires the actuator to follow a moving load by maintaining zero torque against it (Fig. 3). We again demonstrate how k, B and I_m effect the maximum frequency, defined as the frequency the load position can oscillate before a prescribed torque error at the load is exceeded. This situation might occur if the goal of the actuator is to keep contact with an object, while maintaining a constant applied force. Note that there is no inertia at the load as its motion is predefined and is not affected by the applied torque.

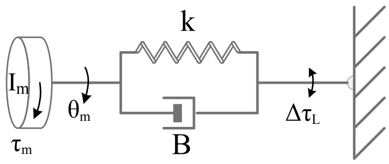


Fig. 2. For the first scenario, the load is fixed to ground ($\theta_L = 0$) while the motor attempts to produce the desired τ_L through the passive dynamic elements k and B.

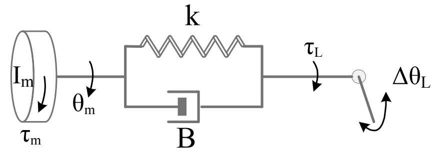


Fig. 3. For the second scenario, the load is forced to move (θ_L) while the motor attempts to keep the load torque (τ_L) zero with the passive dynamic elements k and B.

To determine the effect of k, B and I_m on the performance of the system in any test scenario, we first solve for the motor torque, τ_m, that produces the desired load torque, τ_L. If τ_m remains below the motor's peak torque limit, the system is able to achieve the desired performance goals.

In most cases, as the frequency of the task increases, the required motor torques increase and eventually meets the peak motor torque limit. The function for the exact motor torque, evaluated with torque limits, becomes the basis for describing the relationships that parameters have on achieving the maximum frequency of each task.

The differential equation describing the motion of the system is defined as:

$$I_m\ddot{\theta}_m = \tau_m - \tau_B - \tau_k \quad (1)$$
$$0 = \tau_B + \tau_k - \tau_L \quad (2)$$

where:

$$\tau_k = k[\theta_m - \theta_L]$$
$$\tau_B = B[\dot{\theta}_m - \dot{\theta}_L].$$

To find the required motor torque we take the Laplace transform of (1) and (2), and solve for for the s-domain equation for the motor torque ($T_m(s)$). With all initial conditions ignored, it is calculated as:

$$T_m(s) = \Theta_L(s)\left(I_m s^2\right) + T_L(s)\left(\frac{I_m s^2 + Bs + k}{Bs + k}\right). \quad (3)$$

(3) describes how the load motion and desired load torque effect the required motor torque, where $\Theta_L(s)$ is the s-domain representation of the load motion and $T_L(s)$ is the s-domain representation of the load torque. With this equation, we can define any motion for the load and a desired load torque and determine the exact requirement for the motor torque. At steady state, this computed motor torque will produce the torque at the load with zero error.

V. CHANGING TORQUE AGAINST A STATIC SURFACE

To evaluate the maximum frequency the actuator can achieve under a given set of values for k, B and I_m, we look at the point where the motor's torque becomes greater than the torque limit. At this point the motor is no longer able to produce the required torque to exactly generate the desired τ_L.

To find the motor torque as a function of time, $\tau_m(t)$, we need to define the motion of the load, $\theta_L(t)$ and the desired load torque, $\tau_L(t)$. For this scenario, we hold the load position constant (Fig. 2). We then define the desired load torque to be a sinusoidal function with some angular frequency, ω, and a fixed amplitude of one (the amplitude can be greater or smaller without effecting the relationships as long as it is less than the torque limit)

$$\theta_L(t) = 0$$
$$\tau_L(t) = sin(\omega t). \quad (4)$$

Taking the Laplace transform of $\tau_L(t)$ gives:

$$T_L(s) = \frac{\omega}{s^2 + \omega^2}. \quad (5)$$

Plugging equation (5) back into (3) and taking the inverse Laplace transform, we find the $\tau_m(t)$ required to produce the $\tau_L(t)$ defined in (4) at steady state ($t \gg 0$):

$$\tau_m(t) = \left(\frac{I_m\omega^3 B}{\omega^2 B^2 + k^2}\right)cos(\omega t) + \left(\frac{\omega^2 B^2 - I_m\omega^2 k + k^2}{\omega^2 B^2 + k^2}\right)sin(\omega t). \quad (6)$$

If we consider the extremes of equation (6), we can begin to draw conclusions about the motor requirements and the relationships between the parameters. One extreme occurs when $B = 0$, and equation (6) simplifies to:

$$\tau_m(t) = \left(1 - \frac{I_m\omega^2}{k}\right)sin(\omega t). \quad (7)$$

Equation (7) implies that if the system has very little or no damping, the only way to reduce the torque requirement, for any frequency, is to increase k or decrease I_m.

Conversely, if the system has very little or no elasticity, such that $k \approx 0$, reducing the torque requirement necessitates an increase in B or a decrease in I_m:

$$\tau_m(t) = \left(\frac{I_m\omega}{B}\right)cos(\omega t) + sin(\omega t). \quad (8)$$

Comparing (7) and (8), we note that as the frequency increases, B has a much greater effect than k on reducing the required motor torque.

The graphs in Fig. 4 show the maximum frequency the system can achieve for a set of parameters k, B and I_m. We arbitrarily set $\tau_{limit} = 10$ for each graph and hold I_m

constant for Fig. 4(a) and Fig. 4(b). The graphs demonstrate the effects of modifying the various parameters of equation (6).

It follows from these equations that increasing stiffness provides higher physical bandwidth for applying varying torques to a fixed load. The equations show that there is an inverse relationship between the maximum frequency and the motor inertia (as shown in Fig. 4(c)). In other words, an increase in k or B will increase the bandwidth but an increase in I_m will decrease the bandwidth.

VI. Zero torque against a moving load

To evaluate the maximum frequency, we start by looking at the point where the torque required of the motor becomes greater than the torque limit. For this task we want to find the motor torque as a function of time, $\tau_m(t)$, for a predefined motion of the load, $\theta_L(t)$ and the desired load torque, $\tau_L(t)$. For this scenario, we hold the load torque constant at zero. We then define the desired load position to follow a sinusoidal function at some angular frequency, ω, and an amplitude of θ_A (Fig. 3)

$$\begin{aligned} \theta_L(t) &= \theta_A sin(\omega t) \\ \tau_L(t) &= 0. \end{aligned} \quad (9)$$

Taking the Laplace transform of $\theta_L(t)$ gives:

$$T_L(s) = \theta_A \frac{\omega}{s^2 + \omega^2}. \quad (10)$$

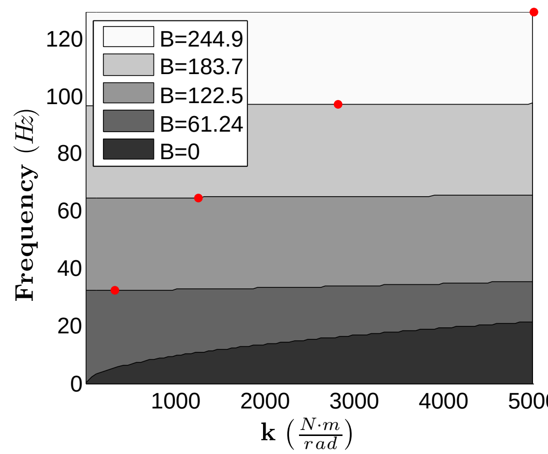
Plugging (10) back into (3) and taking the inverse Laplace transform we find the $\tau_m(t)$ required to produce the $\tau_L(t)$ defined in (9) at steady state ($t \gg 0$):

$$\tau_m(t) = \left(-\theta_A I_m \omega^2\right) sin(\omega t). \quad (11)$$

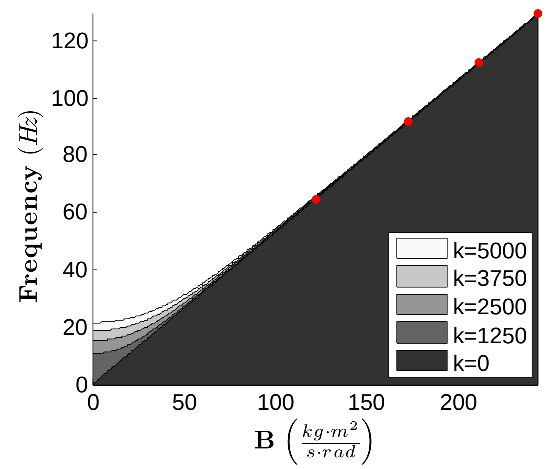
Intuitively this shows that for the motor to exactly produce zero torque at the load, it would have to generate a torque that would cause the motor position (θ_m) to exactly follow the load position (θ_L). We can also conclude that k and B do not matter when trying to follow the load motion. Instead, the only parameter we have for reducing the motor torque requirement is the motor inertia. However it may be more useful to measure the load torque within some error tolerance.

To actually investigate how k and B effect the system, we now assume that there can be error with producing zero torque at the load. To produce an error, we take the optimal output defined in (11) and clip it when the torque limits are encountered.

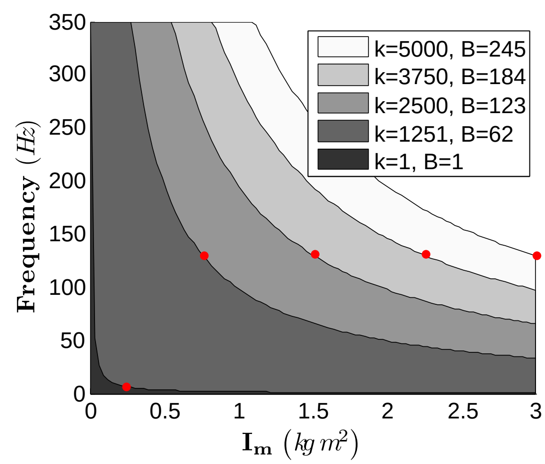
With the limited τ_m as the input, we can find the response at τ_L. This new response will produce an error for which we can choose a threshold based on system requirements. We can now use the error threshold as a metric for defining the maximum frequency the actuator can provide zero τ_L with some acceptable error. What is more important is the response now depends on k and B. Fig. 5 shows an example of how τ_L responds to a limited τ_m.



(a) : Frequency achieved Vs. elasticity, k. Increasing the elasticity slowly increases the maximum frequency.

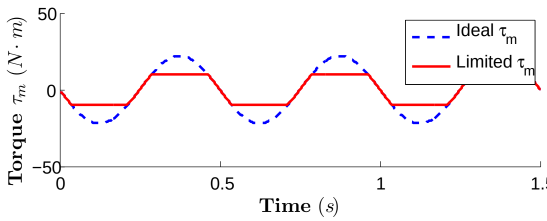


(b) : Frequency achieved Vs. damping, B. Increasing the damping increases the maximum frequency.

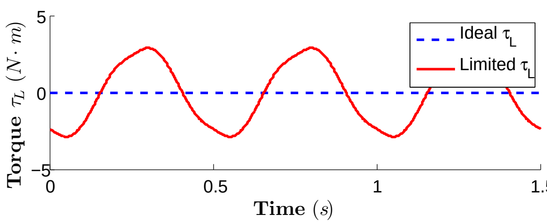


(c) : Frequency achieved Vs. motor inertia, I_m. Increasing the inertia greatly decreases the maximum frequency.

Fig. 4. Performance of a series/damped actuator applying a changing torque against a static surface. The maximum frequency is determined as the point where the load torque error exceeds 0. For reference, the dots on each figure are where the system is critically damped. For figures 4(a) and 4(b), $I_m = 3$.



(a) The input torque, τ_m. The ideal input represents what is needed to produce zero error. The limited input results from the torque limit being applied to the ideal input.



(b) The resulting load torques from the inputs in Fig. 5(a). Notice how the limted τ_m no longer generates zero torque at the load.

Fig. 5. Example responses at the load torque, τ_L, to an the ideal and limited τ_m for applying zero force against a moving load. $I_m = 0.4$, $k = 10$, $B = 1$ and $\tau_{limit} = 10$.

To gain an understanding of how the actuator responds with different parameters, we present the graphs in Fig. 6. Notice that in this scenario, the maximum achievable frequencies quickly become relatively low even with modest values of k and B (Fig. 6(a) and Fig. 6(b)).

These graphs highlight the result that decreasing stiffness provides higher physical bandwidth for tracking the load motion, while maintaining zero load torque. It also shows that there is an inverse relationship between the maximum frequency, f_{max}, and the parameters, k, B and I_m. In other words, a decrease in k, B or I_m will increase the bandwidth.

Even as the stiffness increases to infinity ($k, B \to \infty$), the actuator will never do worse than:

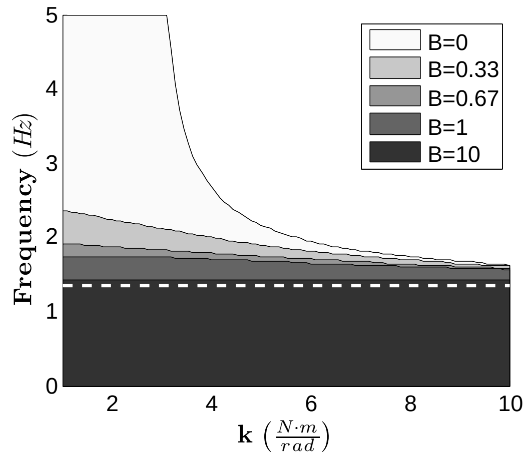
$$f_{worst} = \left(\frac{1}{2\pi}\right) \sqrt{\frac{\tau_{limit}}{\theta_A I_m}}. \quad (12)$$

Equation (12) is derived by setting equation (11) equal to τ_{limit} and solving for the frequency.

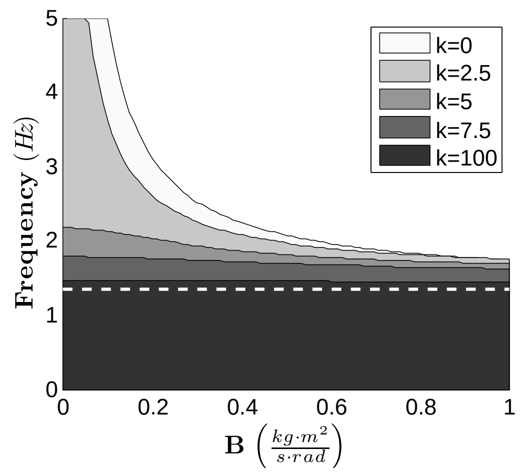
The frequency f_{worst} represents the maximum frequency the load motion can move at before the motor torque limit, τ_{limit}, is reached. For any frequency beyond f_{worst} there will be an error, whose magnitude depends on the inertia of the motor. This threshold is plotted as the dashed white line in Fig. 6(a) and Fig. 6(b).

VII. Conclusions and Future Work

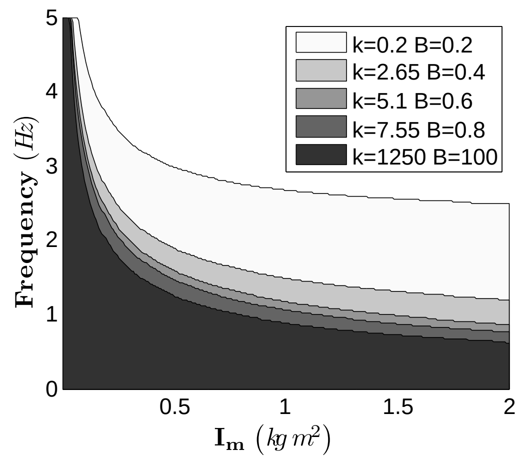
This paper derives the fundamental limitations of the actuator in Fig. 1 with various passive dynamics. Our model does not include active control; we computed the optimal input to the system to produce the required torque at the load with zero error. This is important so that our results only reflect the physical system's performance.



(a) : Frequency achieved Vs. elasticity, k. Increasing the elasticity decreases the maximum frequency.



(b) : Frequency achieved Vs. damping, B. Increasing the damping decreases the maximum frequency.



(c) : Frequency achieved Vs. motor inertia, I_m. Increasing the motor inertia decreases the maximum frequency.

Fig. 6. Performance of a series/damped actuator applying Zero force against a moving load. The maximum frequency is determined as the point where the load torque error exceeds 1 $N \cdot m$. The white dashed lines in figures 6(a) and 6(b) represent the maximum frequency as the system stiffness approaches infinity and is the worst case. For figures 6(a) and 6(b), $I_m = 0.4$.

This paper develops relationships between an actuator's passive dynamics (k, B, I_m and τ_{limit}), for the purpose of better understanding how to select parameters for a force-control task. We present two distinct scenarios for the actuator system and calculate the required input to produce the desired output with zero error. These exact solutions provide the basis for understanding how the parameters effect bandwidth.

For our model to generate a varying torque against a fixed load, the system should have higher stiffness and/or lower inertia. Perhaps less clear is that both damping and inertia play a much larger role in increasing the maximum frequency than stiffness.

For the actuator to produce exactly zero torque against a moving load, the system's stiffness does not matter. Instead, the stiffness only defines how quickly the error increases with increased frequency. This paper showed that by reducing the system stiffness, the error caused by motor torque limits is reduced. But as the stiffness approaches infinity, the performance of the actuator is governed solely by the motor inertia and torque limit. It is evident from the results that designing an actuator that is optimized at both tasks is very difficult. The only way to improve bandwidth in both cases is to reduce the motor inertia or increase the torque limit.

Additional studies will include the development of relationships for more complex actuation scenarios such as stopping an inertia or mass with initial velocity or commanding the actuator to behave like a specific spring. These tasks should help enlighten us on the roles of elasticity and damping beyond determining if we need more or less of either.

The final step in our work is to validate the calculations presented on a real system. We have begun constructing an actuator that embody the model presented in this paper (Fig. 7).

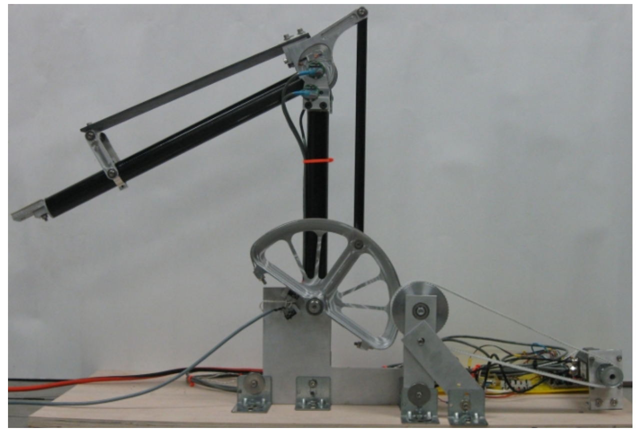


Fig. 7. Test platform for a single degree of freedom force controlled actuator. The system measures the force applied by measuring the deflection in the spring. Different springs of widely varying stiffness can quickly be exchanged.

VIII. ACKNOWLEDGMENTS

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