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EFFECTS OF LEG CONFIGURATION ON RUNNING AND WALKING ROBOTS*

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This paper investigates the relations between leg configuration and performance during running and walking operations. Specifically, we use standard kinematic techniques to visualize impact forces and kinetic energy loss during inelastic collisions with the environment in order to gain fundamental insights into robotic leg design. We show that for a two-link planar leg with revolute joints, a slightly crouched stance with a backward-facing knee (in contrast to humans) is optimal for both impact rejection and energy economy.

Keywords: legged, locomotion, collisions, dynamic, running

1. Introduction

Legged robots intended for energetic and dynamic locomotion encounter a repetitive series of collisions with the ground during every stride as the robot transitions from flight phase to stance phase. These collisions become especially prevalent in dynamic, unknown, or treacherous environments where unexpected and larger-than-normal impact forces often occur. At walking speeds, the severity and frequency of collisions can be minimized through knowledge of the environment and cautious control strategies, however at greater speeds, collisions become unavoidable due to the ballistic nature of running gaits. Assuming the leg actuators have sufficient bandwidth or series impedance to isolate actuator inertia, such collisions still have several undesirable traits as a result of leg inertia, including energy loss due to the

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inelastic nature of the collisions and large end-effector impulses which could result in mechanical damage or failure.

We contend that the severity of these inelastic collisions with the environment can be minimized by strategically choosing the leg configuration during impact. The primary motivation for this work stems from our observations of the monopod robot Thumper, which showed a distinct preference for backward-hopping (in contrast to humans).[1] During stable backward-hopping experiments, Thumper was more energy economic, produced less ground impact noise, and was less susceptible to falling as compared to forward-hopping. Building on these observations, we investigate the implications of leg configuration on running and walking operations. Specifically, we find that for a two-link planar leg with revolute joints, a slightly crouched stance with a backward-facing knee (in contrast to humans) is optimal for both impact absorption and energy economy.

2. Methods

In order to quantitatively evaluate the kinetic energy loss and forces incurred during an impact with the environment, we borrow techniques commonly used for robotic manipulator analysis.[2] Note that the expressions given by Eq. (1) through Eq. (5) are standard in dynamics literature; they are provided here for completeness.

2.1. *Joint Space Kinematics*

We begin by reviewing the forward kinematic relationships between the joint velocities $\dot{q}$ and the corresponding end-effector velocity

$$\dot{x} = [J(q)]\dot{q}, \tag{1}$$

where $J(q)$ is the Jacobian matrix associated with the coordinate transformation from cartesian end-effector velocity to rotational joint velocities. Because we are interested in changes of kinetic energy, we proceed by considering the inverse kinematics problem relating the instantaneous change in the end-effector velocity $\Delta\dot{x}$ to the instantaneous change in the joint velocities

$$\Delta\dot{q} = \bar{J}(q)\Delta\dot{x}, \tag{2}$$

where $\bar{J}(q)$ is the dynamically consistent generalized inverse of the Jacobian matrix associated with the solution that minimizes the instantaneous kinetic energy.[3,4] For non-redundant manipulators, $\bar{J}(q)$ is simply the inverse of the Jacobian, however, for redundant manipulators

$$\bar{J}(q) = H^{-1}(q)J^T(q)[J(q)H^{-1}(q)J^T(q)]^{-1}, \tag{3}$$

where $H(q)$ is the symmetric positive-definite inertia tensor of a series of rigid bodies with respect to the joint coordinates. Substituting Eq. (3) into Eq. (2), allows us to determine the instantaneous change in joint velocities associated with a given instantaneous change in end-effector velocity.

2.2. *Kinetic Energy Loss*

For the purposes of this paper, we assume that collisions are perfectly inelastic, with no slip, in which neither the robot nor the environment deforms at the contact point. This is a reasonable assumption as we want the end-effector to remain planted on the ground to prevent chatter and slip. By using this criterion and representing the environment as a infinitely massive, immovable object with zero velocity, we can deduce that the end-effector velocity after impact becomes zero. We can investigate the kinetic energy loss ΔT associated with this instantaneous change in velocity through

$$\Delta T = \frac{1}{2}\Delta\dot{q}^T H(q)\Delta\dot{q}. \tag{4}$$

Because the instantaneous change in joint velocities is not typically known succeeding a collision, we choose to express the kinetic energy loss corresponding to an instantaneous change in end-effector velocity. Substituting Eq. (2) into Eq. (4) yields

$$\Delta T = \frac{1}{2}\Delta\dot{x}^T \bar{J}(q)^T H(q)\bar{J}(q)\Delta\dot{x}, \tag{5}$$

where $\bar{J}(q)^T H(q)\bar{J}(q)$ is denoted as the symmetric generalized inertia tensor of a series of rigid bodies $G(q)$.[5,6] Equation (5) allows us to conveniently calculate the kinetic energy loss of a manipulator given a instantaneous change in end-effector velocity.

3. Analysis

The primary goal of this investigation is to not only evaluate the performance of a particular leg configuration, but also to quantitatively and visually compare different configurations. Doing so will allow us to gain fundamental insights into robotic leg design and develop an intuition for why one configuration may be superior to others.

3.1. *Two-Link Model*

While the methods presented in this paper are applicable to a wide range of systems, we will focus on a particular model, the two-link planar manipulator with revolute joints. As illustrated in Fig. 1, the system consists of two rigid bodies hinged together by a pin joint allowing for two degrees of freedom relative to a fixed hip. In order to determine the kinetic energy

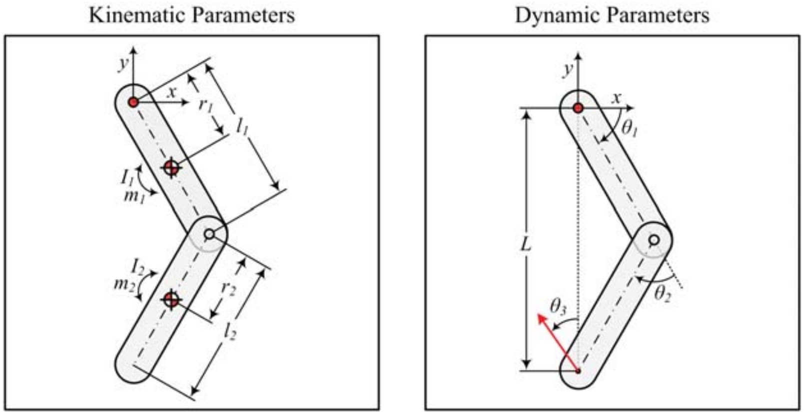

Fig. 1. Illustration of the kinematic and dynamic parameters for the two-link planar manipulator with revolute joints. Left: mass, inertia and dimensional constants are depicted. Right: the joint coordinates are shown relative to the fixed coordinate system at the hip, the impact vector (solid arrow) is shown extending from the toe.

loss associated with a configuration, we first need to derive the kinematic relationships for the two-link model. We proceed by deriving the inertia tensor with respect to the joint coordinates from Lagrange's equations.

$$H(q) = \begin{bmatrix} \alpha + 2\beta c_2 & \delta + \beta c_2 \\ \delta + \beta c_2 & \delta \end{bmatrix} \tag{6}$$

where

$$\alpha = I_{z1} + I_{z2} + m_1 r_1^2 + m_2(l_1^2 + r_2^2) \tag{7}$$

$$\beta = m_2 l_1 r_2 \tag{8}$$

$$\delta = I_{z2} + m_2 r_2^2 \tag{9}$$

Additionally, we determine the Jacobian matrix by differentiating the forward kinematic mapping.

$$J(q) = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix} \tag{10}$$

where $s_i = \sin\theta_i$, $s_{ij} = \sin\theta_{i+j}$, and similarly for c_i and c_{ij}.

From the illustration on the right in Fig. 1, we can see that every configuration is represented by the unique combination of three parameters; θ_1 and θ_2, which determine the shape and orientation of the leg, and θ_3, which represents the impact angle relative to the generalized coordinate system. For convenience, the problem is simplified recognizing that θ_1 and θ_2 are dependent on the virtual leg length L due to the non-redundant nature of the two-link model. By then aligning the generalized coordinate system to the virtual leg length, we can essentially eliminate one degree of freedom and more concisely describe the system with only two parameters, L and θ_3. This allows us to independently visualize the effects of leg configuration and impact angle on performance.

3.2. *Kinetic Energy Loss*

Using the formulation in Eq. (5), the kinetic energy loss is calculated relative to the leg length and impact angle, generating the contour surface depicted in Fig. 2. The prominent feature of this plot is the valley starting at $\theta_3 = 0$, $L = 0$ stretching up to $\theta_3 = \pi/2$, $L = 1$. This region represents the optimal combinations of leg lengths and impact angles for the given kinematic parameters. In this case, the upper and lower leg segments are equally proportioned with evenly distributed mass throughout.

4. Discussion

The results shown in Fig. 2 reveal several fundamental insights into robotic leg design as it pertains to dynamic locomotion.

4.1. *Impact Angle*

During a typical running gait, the foot is extended in front of the body, and the resulting impact with the ground occurs predominately in the vertical direction with a small horizontal component. As illustrated in Fig. 3, by rotating the system as to align the impact vector with the vertical, two unique cases appear.

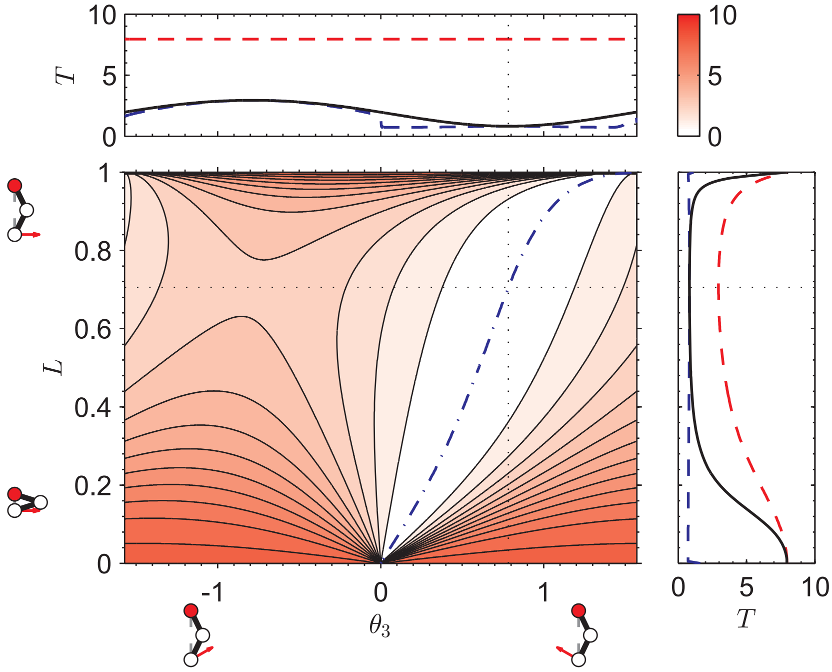

Fig. 2. The contour plot illustrates kinetic energy loss T as a function of virtual leg length L and impact angle θ_3 with the optimal (minimum energy loss) at varying leg lengths indicated by the dot-dashed line within the white shaded region. The top and side panels present the kinetic energy loss (solid black lines) through a two-dimensional slice (dotted black lines) of the three-dimensional contour surface. For a qualitative comparison, the minimum and maximum kinetic energy loss (dotted lines) along each dimension are also provided. (Because of end-effector singularities, the magnitudes of the energy loss function are scaled by an arctangent function.)

Assuming the leg is extended in front of the body we deduce the direction of travel for each case. We can see that values of $\theta_3 < 0$ correspond to a forward-facing knee (as compared to humans), while values of $\theta_3 > 0$ correspond to a backward-facing knee (in contrast to humans). Examining these two regions in Fig. 2, we can see that the vast majority of the low energy loss area (depicted as the filled white area) falls in the region of $\theta_3 > 0$. This is an interesting result that suggests a backward-facing knee is optimal for impact rejection and energy economy.

This can be intuitively explained by the difference in the generalized inertia matrices. With a backward-facing knee, the toe impact causes an

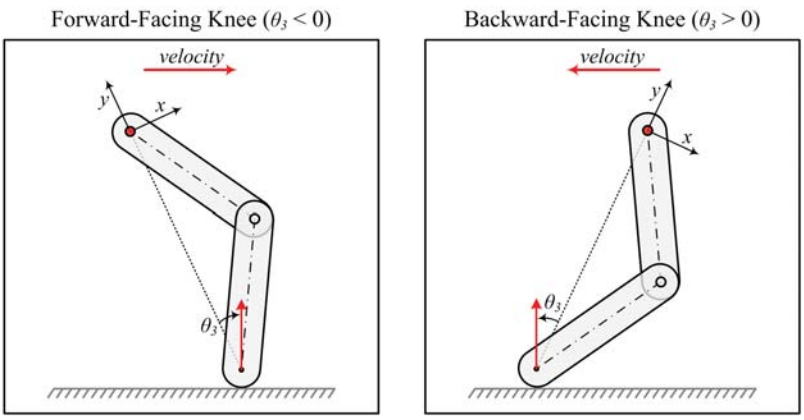

Fig. 3. Illustrations of the two unique leg configurations associated with a typical running gait impact. Left: the case of $\theta_3 < 0$, analogous to a forward-facing knee (as compared to humans). Right: the case of $\theta_3 > 0$, analogous to a backward-facing knee (in contrast to humans).

instantaneous rotation of the shin while a forward-facing knee results in an instantaneous translation of the shin, which, requires a greater impulse. This is in direct alignment with our observations made of the monopod robot Thumper which exhibited a preference for backward-hopping.[1]

4.2. *Leg Configuration*

The effect of leg configuration on kinetic energy loss is substantial as observed in Fig. 2. The optimal virtual leg length for this particular case that guarantees the lowest energy loss independent of impact angle occurs at $L = 0.71$ (as indicated by the dotted horizontal line in the side panel of Fig. 2). Leg lengths in close proximity to this optimal value ($L = 0.5$ to 0.9) show only slight increases in energy loss while lengths further away result in drastically increased energy loss. We can see that configurations in the vicinity of a singularity result in the largest energy loss and thus also the largest impact forces. This is especially true when the impact angle is aligned with virtual leg length. These findings agree with natural intuition and experience; when an impact is imminent we often slightly bend our knees or elbows to reduce risk of injury. This result suggests that a slightly crouched stance allows for better impact rejection and energy economy.

5. Conclusions

In this paper, we discussed the effects of leg configuration on running and walking robots from an impact perspective. We conclude that during walking and running operations, a slightly crouched posture allows large impact forces to be more readily rejected with minimal energy loss. We can also show that force generation at the end-effector can be maximized by extending the leg joints towards a singularity which may be beneficial in static situations such as standing. More interesting, is the difference between forward and backward running configurations. We show that a backward-facing knee (in contrast to humans) more efficiently rejects impacts then a forward-facing knee (as compared to humans). These results are intuitive yet have significant implications in robotic leg design.

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